A New Integral Variable Structure Controller
For Incorporating Actuator Dynamics

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Abstract

In this paper, a new simple integral variable structure controller is designed with incorporating the actuator dynamics. The formulation of the VSS(variable structure system) controller design includes integral augmented sliding surface and the dynamics of the actuator expressed as the state equation. An illustrative example is given to show the effectiveness of the developed controller.

Key words: Actuator dynamics, sliding mode control, variable structure system, integral sliding surface

I. Introduction

The VSS is classified into the robust, discontinuous, nonlinear, and deterministic control categories. This algorithm can provide the outstanding means for controlling the uncertain plants[1]. Since the mathematical development of the VSS(variable structure system) by Filippov in 1960[2], the research of the VSS has been mature now because of the development of the general design method for wide spectrum of system types and applications to variety control problems such as stabilization, regulation, tracking, and even identification[3]–[6].

For implementing the control inputs of plants, generally, a certain type of actuators is used. If the dynamics of actuator is low such as in case of large scale systems, it is necessary to incorporate the dynamics of actuators into a design of controllers[7]. As one example of the large scale system, the dynamics of airplanes is highly nonlinear complex and incorporating the first order dynamics of the actuators for flight controls. Until now in general control area, the nonlinearity property of actuators saturation is considered in the design of controllers in view of the stability and stable gain design[8]. In the research area of the VSS, it is difficult to find the research results incorporating actuator dynamics[9]–[13].

In this paper, an integral VSS incorporating the actuator dynamics is briefly designed for the first time in the VSS research area. The dynamics of actuators is expressed as a state equation. The design of the integral VSS controller is formulated after combing the plant dynamics with actuator one. The integral sliding surface is adapted. An illustrative example is given to show the usefulness of the designed algorithm.

II. A New Valuable Structure Controller

2.1 Description of Plants and Actuators

A multi-input uncertain dynamical plant with actuators can be separately described in state equation as [7]

\[
\dot{X}_p = (A_p + \Delta A_p)X_p + (B_p + \Delta B)X_u + D_p(t) \\
\dot{X}_u = (A_u + \Delta A_u)X_u + B_u U
\]

(1)

where \(X_p \in \mathbb{R}^n\), \(X_u \in \mathbb{R}^m\), and \(U \in \mathbb{R}^m\) are the state variables of plants, the state variable of corresponding actuators, and control input, respectively. It is assumed that the uncertain \(\Delta A_p\), \(\Delta B_p\), \(\Delta D_p\), and \(\Delta A_u\) are bounded and satisfy

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following the matching condition
\[ \Delta A_{p} + \Delta B_{p} + D_{p} \in \mathbb{R}(B_{p}) \]
\[ \Delta A_{s} \in \mathbb{R}(B_{s}) \]  
(2)
The first equation of (1) stands for the dynamics of the plants controlled by the actuator state. The second state equation of (1) describes the dynamic behaviors of the actuator devices for newly incorporating into the design of the VSS controller. Most of the practical actuators are implemented independently, which means that $B_{s}$ has a diagonal structure. Combing both plant and actuator dynamics leads to
\[ \dot{X} = (A + \Delta A)X + (B + \Delta B)U + D(t) \]  
(3)
where
\[ A = \begin{bmatrix} A_{p} & B_{p} \\ 0 & A_{s} \end{bmatrix}, \quad \Delta A = \begin{bmatrix} \Delta A_{p} & \Delta B_{p} \\ 0 & \Delta A_{s} \end{bmatrix} \]
\[ B = \begin{bmatrix} 0 \\ B_{s} \end{bmatrix}, \quad D(t) = \begin{bmatrix} D_{p}(t) \\ 0 \end{bmatrix} \]  
(4)
\[ \dot{X}_{s} = -[C_{p} C_{s}]^{-1} [C_{p} C_{sa}] X_{s} \]  
(9)
where $X_{s}$ is the solution of (9) and identically implies a state set of the ideal sliding surface. Now, by directly applying the well-developed feedback theoreists the ideal sliding dynamics (9), for example eigenstructure assignment or optimal control, the coefficient matrix of the surface (7) or (9) can be chosen effectively.

Then, as the second design step, the corresponding control input for stabilizing the sliding surface to be zero is suggested with composing of the continuous term, discontinuous term, and continuous feedback of the switching surface as
\[ U = K_{x} \cdot \dot{X}_{s} - K_{eq} \cdot X_{s} - \Delta K \cdot \dot{X}_{s} \]  
(10)
where each gain designed as follows:
\[ K_{x} = [C_{p} B_{p}]^{-1} [C_{p} B_{pa}] \]  
(11)
\[ K_{eq} = [C_{s} B_{s}]^{-1} [C_{s} A_{p} + C_{sp} C_{s} B_{p} + C_{sa} + C_{sa}] \]
\[ \Delta K = [C_{s} B_{s}]^{-1} [\Delta K_{p} \Delta K_{s}] \]  
(13)

where $X_{s}$ is the combined reference command value for the plant and actuator as
\[ X_{s} = \begin{bmatrix} X_{p}^{r} \\ X_{p}^{r} \end{bmatrix} \]
(8)
where $X_{p}^{r}$ and $X_{a}^{r}$ are the reference values for the plant and the actuator, respectively. By introducing the initial condition and integral term into the right hand side of (6), it is possible to make the switching surface be zero at $t=0$ so that there is no reaching phase problems. In (6), constant coefficient matrices $C_{p}$, $C_{s}$, $C_{sp}$, and $C_{sa}$ are the design parameters with the condition $\det(C_{p} B_{p}) \neq 0$. For systematic design of the integral surface (6), the ideal sliding surface is derived in the dynamic form from $\dot{S}(t) = 0$ as
\[ \dot{X}_{s} = -[C_{p} C_{s}]^{-1} [C_{p} C_{sa}] X_{s} \]  
(9)
where $X_{s}$ is the solution of (9) and identically implies a state set of the ideal sliding surface. Now, by directly applying the well-developed feedback theoreists the ideal sliding dynamics (9), for example eigenstructure assignment or optimal control, the coefficient matrix of the surface (7) or (9) can be chosen effectively.

2.2 Design of Proposed VSS

For the plant (3), the suggested VSS will be designed through two steps, determination of the switching surface and choice of corresponding control function to stabilize the system on the surface. First, the integral sliding surface is proposed in the simplest form for incorporating the dynamics of actuators as
\[ S(t) = C_{p} [X_{p}(0) - X_{p}(t)] + C_{p} [X_{p}(0) - X_{p}(t)] + [C_{sp} C_{sa}] \int_{0}^{t} (X - X(r)) \, dr \]  
(7)
where $X_{s}$ is the combined reference command value for the plant and actuator as
\[ X_{s} = \begin{bmatrix} X_{p}^{r} \\ X_{p}^{r} \end{bmatrix} \]
(8)
\[ \dot{X}_{s} = -[C_{p} C_{s}]^{-1} [C_{p} C_{sa}] X_{s} \]
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Then, as the second design step, the corresponding control input for stabilizing the sliding surface to be zero is suggested with composing of the continuous term, discontinuous term, and continuous feedback of the switching surface as
\[ U = K_{x} \cdot \dot{X}_{s} - K_{eq} \cdot X_{s} - \Delta K \cdot \dot{X}_{s} \]
(10)
where each gain designed as follows:
\[ K_{x} = [C_{p} B_{p}]^{-1} [C_{p} B_{pa}] \]
(11)
\[ K_{eq} = [C_{s} B_{s}]^{-1} [C_{s} A_{p} + C_{sp} C_{s} B_{p} + C_{sa} + C_{sa}] \]
(12)
\[ \Delta K = [C_{s} B_{s}]^{-1} [\Delta K_{p} \Delta K_{s}] \]
(13)

where $X_{s}$ is the combined reference command value for the plant and actuator as
\[ X_{s} = \begin{bmatrix} X_{p}^{r} \\ X_{p}^{r} \end{bmatrix} \]
(8)
where $X_{p}^{r}$ and $X_{a}^{r}$ are the reference values for the plant and the actuator, respectively.
\[ K_{x_{1i}} > \max |D_{1}(t)| \]  \hspace{1cm} (16)

\[ K_{2j} > 0 \]  \hspace{1cm} (17)

\[ i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \]

The constant gains, (11) and (12), are directly determined according to the choice of the integral sliding surface, the gains from (13) to (17) are the design parameters in the present control input, specially (13) is discontinuous term. And (16) and (17) are the constant gains for continuous feedback of the switching surface. Until now, the integral VSS for incorporating the dynamics of the actuators is presented for the first time. The derivative of the integral sliding surface as the dynamics of the surface resulted from the above control input is derived from \( S(t) = 0 \) as

\[ \dot{S}(t) = C_p [\dot{X}_p(0) - \dot{X}_p(t)] + C_q [\dot{X}_q(0) - \dot{X}_q(t)] + C_a [\dot{X}_a(0) - \dot{X}_a(t)] \]

\[ + [C_{Dp} C_{Da} (X_s - X(t))] \]

\[ = -(C_a A_p + C_{aq}) X_p - (C_p B_p + C_{ap} A_p + C_{aq} A_q) X_q + (C_p C_b X_q + C_{aq} X_a - C_b B_p U - C_{lp} X_p - C_{lp} B_p U) - (C_{lp} A_p + C_{lp} A_q) X_a - C_{lp} D_p(t) \]  \hspace{1cm} (18)

By applying the above control input into (18), the dynamics of the sliding surface is simply rearranged as

\[ \dot{S}(t) = C_q \Delta A_p X_p - (C_a \Delta B_p + C_a \Delta A_q) X_q \]

\[ - C_q B_p(t) \cdot \text{TRIANGLE} \dot{X}_q - \text{TRIANGLE} K_s X_q \]

\[ - K_{s1} S(t) - K_{s2} \frac{S^{2}}{|S| + \delta} \]  \hspace{1cm} (19)

2.3 Stability Investigation

To prove the stability of the closed system and the existence condition of the sliding mode, take a Lyapunov candidate function as

\[ V(t) = \frac{1}{2} S^T(t) \cdot S(t) \]  \hspace{1cm} (20)

Differentiating (20) with respect to time and from (19), the derivative of (20) becomes

\[ \dot{V}(t) = S^T(t) \cdot \dot{S}(t) = \sum_{i=1}^{m} \delta_i(t) \cdot \dot{s}_i(t) \]

\[ = C_q \Delta A_p X_p S + (C_a \Delta B_p + C_a \Delta A_q) X_q S - C_q B_p(t) S \]

\[ - [\Delta K_s X_q S + \Delta K_q X_q S + K_{s1} S^2 + K_{s2} \frac{S^2}{|S| + \delta}] \]  \hspace{1cm} (21)

Substituting (13)–(17) into (21), (22) can be obtained

\[ \dot{V}(t) < 0 \text{ and } S_i \cdot \dot{S}_i, \quad i = 1, 2, \ldots, m \]  \hspace{1cm} (22)

when which completes the proof the stability of the closed system and the existence condition of the sliding mode at the same time.

From the results of the stability proof, the suggested control input (10) with the gain (11)–(17) can stabilize the integral sliding surface (6), to be zero.

III. Example

3.1 Description of Plant and Actuator

To show the effectiveness of the algorithms simple, an example of the multimode flight control problems[7] is considered. The simple flight dynamics of aircraft and actuator is separately described as

\[ X_{p} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0.8663 & 42.233 \end{bmatrix} \cdot X_{p} + \begin{bmatrix} 0 \\ 0 \\ 0.9933 \\ -1.341 \end{bmatrix} \]

\[ \begin{bmatrix} 0 & -17.251 & -1.5766 & -0.1689 & -0.2518 \end{bmatrix} \]

\[ X_{a} = \begin{bmatrix} 20 \\ 0 \\ -20 \end{bmatrix} X_{p} + \begin{bmatrix} 20 \\ 0 \\ 20 \end{bmatrix} X_{a} \]  \hspace{1cm} (23)

where each variable is defined as

\[ X_{p} = \begin{bmatrix} \theta \\ q \\ \alpha \end{bmatrix} \]

- \( \theta \) - pitch attitude

- \( q \) - pitch rate

- \( \alpha \) - angle of attack

\[ X_{a} = \begin{bmatrix} \delta_{e} \end{bmatrix} \]

- \( \delta_{e} \) - elevator deflection command

\[ U = \begin{bmatrix} \delta_{fl} \end{bmatrix} \]

- \( \delta_{fl} \) - flap deflection command  \hspace{1cm} (24)

The equation of (23) is the most simple longitudinal dynamics of aircraft and the equation of (24) is the corresponding actuator dynamics. The longitudinal dynamics of aircrafts pitch attitude, pitch rate, and angle of attack are controlled by the actuators, elevator and flaperon[7].

3.2 Design if VSS

For the pitch pointing mode(PPM) control problem, a command of the flight attitude is given as

\[ X_{pr} = \begin{bmatrix} 2^\circ & 0 & 2^\circ \end{bmatrix} \text{RIGHT} \]  \hspace{1cm} (26)

and for the vertical translation mode(VTM) control problem, a command of the flight attitude is given as

\[ X_{pr} = \begin{bmatrix} 0 & 0 & 2^\circ \end{bmatrix} \text{RIGHT} \]  \hspace{1cm} (27)

Using the ideal sliding dynamics (8), the design parameters for the integral sliding surface (6) are selected as

\[ C_p = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 \end{bmatrix} \text{RIGHT} \], \quad C_q = \begin{bmatrix} -0.3 & 0 \\ 0 & -0.1 \end{bmatrix} \text{RIGHT} \]
\[ C_{yp} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_{oa} = 0_{2 \times 2} \]  
(28)

and the chosen gain for the control inputs are as follows:

\[ K_s = \begin{bmatrix} -0.1667 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0 \end{bmatrix} \]  
\[
K_{q} = \begin{bmatrix} -0.1667 & -0.5218 & -7.2038 & 1.8752 & 0.2628 \\ 0 & -1.9866 & 2.1820 & 0.3378 & -0.4964 \end{bmatrix} \]  
(29)

\[ \text{TRIANGLEK} = 0_{2 \times 5}, \quad K_{s1} = K_{s2} = 0_{2 \times 2} \]

The results of the simulations are depicted in Fig.1 for the pitch pointing mode(PPM) control problem and Fig. 2 for the vertical translation mode(VTM) control problem. As can be shown, each state output is exactly regulated to given command \( X_{pr} \) and each actuator state well follows given each control command. The two sliding surfaces are regulated to near zero.

**IV. Conclusions**

In this paper, a new simple integral variable structure controller incorporating the dynamics of actuators is designed. In formulation of the VSS design, the dynamics of the actuator in form of state equation is included. After the reference command for the actuator is obtained, the integral sliding surface without the reaching phase problems is suggested in error coordinate of the states of the plant and actuator. The corresponding control input is presented in order to generate the sliding mode at every point on the suggested surface. The closed loop stability and existence condition of the sliding mode is proved in detail. Through simulation studies on the flight controls, the usefulness is shown effectively. The presented control technology can be applicable to the control problem where the actuator dynamics is not much faster rather than that of the plant.

**References**


Fig. 1. Pitching pointing mode \( x_p = [\theta, 0, \zeta] \)

Fig. 2. Vertical translational mode \( x_v = [0, 0, \zeta] \)


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**BIOGRAPHY**

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